Decoherence from spin environment: Role of the Dzyaloshinsky-Moriya interaction

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(Received 4 April 2009; published 15 May 2009)

We study the time evolution of decoherence factor of two spin qubits and qutrits coupled to an *XY* spin chain with Dzyaloshinsky-Moriya (DM) interaction environment. The dynamical process of the decoherence is numerically and analytically investigated. We find that the DM interaction can enhance slightly the decay of the decoherence factor in the weak-coupling region. However, in the strong-coupling region, the decoherence factor is very sensitive to the DM interaction.

DOI: 10.1103/PhysRevA.79.052320 PACS number(s): 03.67.Mn, 03.65.Yz

I. INTRODUCTION

As a fundamental concept in quantum theory, quantum entanglement plays a key role as a potential resource for quantum information processing (QIP) and attracts much attention in many branches of physics both in theoretical aspects and experimental ones [1–3]. On the other hand, a realistic system is surrounded by an environment, and the unavoidable coupling between them will lead to decoherence of the entanglement [4,5], which is a major obstacle for building a practical quantum computer. This complex quantum many-body phenomenon is an interesting problem and many works have been dealt with various models [6–19]. Yu and Eberly [7] found that two entangled qubits become completely disentangled in a finite time under the influence of pure vacuum noise. Dodd and Halliwell [8] studied the competing effects of environmental noise and interparticle coupling on disentanglement by solving the dynamics of two harmonically coupled oscillators. Tsomokos *et al.* [16] studied the dynamics of entanglement in qubit chains influenced by noise and static disorder and they found that the amount of disorder that is typically present in experiments does not affect the entanglement dynamics dramatically, while the presence of noise can have a significant influence on the generation and propagation of entanglement. Oxtoby *et al.* [17] investigated the effects of the spin-boson environment on a double-qubit register initially prepared in a maximally entangled state and revealed that the presence of TLF-TLF (two-level fluctuators) coupling reduced the performance of entangling gate operations performed on the register. Recently, there is a growing interest in the study of decoherence due to spin baths [20–22]. Cucchietti *et al.* [21] examined the decoherence of a central spin interacting with a collection of environment spins. They revealed that the decoherence factor will generically have a Gaussian decay when there is no self-Hamiltonian for the system and the main feature of the decoherence process can be dramatically changed by adding a self-Hamiltonian for the system. For a correlated spin environment, Rossini *et al.* [22] analyzed a model of environment consisting of an interacting one-dimensional quantum spin-1/2 chain and they revealed that at short time the Loschmidt echo (LE) decays as a Gaussian, while for long time it approaches an asymptotic value which strongly depends on the transverse field strength. Furthermore, some researchers investigated the decoherence of a system coupled to a spin environment with quantum phase transition (QPT) [23–29], which happens at zero temperature and is driven only by quantum fluctuation [30]. At the critical point there usually exists degeneracy between the energy levels of the system. For a quantum two-level system coupled to an Ising chain, Quan *et al.* [23] found that the decaying behavior of the LE is best enhanced by the QPT of the surrounding system. Cucchietti *et al.* [24] discovered that the decoherence induced by the critical environment possesses some universal behavior. For a two-qubit case under a transverse Ising model, Sun *et al.* [26] found that the concurrence decays exponentially with fourth power of time in the vicinity of the critical point of the environment system. The multiqubit case was studied by Ma *et al.* [28].

Recently, several experiments evidenced that entanglement can exist in variety of macroscopic and strong correlated systems in solid-state matter [31] and some magnetic compounds can be used as an entanglement bath [32]. The Dzyaloshinsky-Moriya interaction is often present in the models of many low-dimensional magnetic materials. Despite being small, this interaction is known to generate many spectacular features [33–35]. In the field of condensed matter physics, antisymmetric spin coupling was first suggested by Dzyaloshinsky [36] to explain the mechanism of weak ferromagnetism of antiferromagnetic crystals from purely symmetry ground state and was later derived theoretically by Moriya [37]. The Dzyaloshinsky-Moriya (DM) exchange interaction is proportional to the vector product of interacting spins and is allowed by symmetry in noncentric crystal structures. In this work, we consider two spins coupled to an *XY* spin chain environment with DM interaction in order to re-

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veal the effect of the DM interaction on the dynamic evolution of the two-spin entanglement. We will investigate the role of DM interaction in the decoherence, given different coupling intensity regions, and both the two-qubit and two-qutrit cases will be considered.

The paper is organized as follows. In Sec. II, we introduce two-spin model system coupled to XY spin chains with DM interaction. By exactly diagonalizing the Hamiltonian, we give an expression of the time evolution operator. In Sec. III, the analytical analysis and numerical computation are carried out to understand the behavior of the decoherence factor both in the weak-coupling region and the strong one. In Sec. IV, a similar analysis to two qutrits coupled the spin chain is given. Finally, we give a summary for our main results.

II. MODEL AND SOLUTION

We choose the environment to be a general XY spin chain with DM interaction and the two spins are transversely coupled to the chain. The corresponding Hamiltonian reads

$$H = J\sum_{l}^{L}\left[\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right] + J\sum_{l}^{L} D(\sigma_l \times \sigma_{l+1}) + \frac{g}{2}J(s_A^z + s_B^z)\sum_{l}^{L}\sigma_l^z = H_E + H_I. \tag{1}$$

Here, we set the spin coupling interaction in Eq. (1), $J=1$, and the parameters D, λ, and g are dimensionless values. H_I denoted by $\frac{g}{2}J(s_A^z+s_B^z)\Sigma_l^L\sigma_l^z$ represents the interaction between the system and the environment. L is the total number of the sites of the environment. Apart from the interaction Hamiltonian H_I, the remaining part of the whole Hamiltonian is the self-Hamiltonian H_E of the environment consisting of an XY spin chain with DM interaction. λ characterizes the strength of the transverse field and g denotes the coupling strength between the chain and the two spins s_A^z and s_B^z. σ_l^α $(\alpha=x,y,z)$ are the Pauli operators defined on the lth site and D denotes the intensity of the DM interaction imposed along the z direction. Noticing that $[s_A^z+s_B^z,\sigma_l^\alpha]=0$, Eq. (1) can be rewritten as

$$H = \sum_{\mu=1}^{\zeta}|\phi_\mu\rangle\langle\phi_\mu| \otimes H_E^{(\lambda_\mu)}, \tag{2}$$

where $|\phi_\mu\rangle$ $(\mu=1,\ldots,\zeta)$ is the μth eigenstate of operator $\frac{g}{2}(s_A^z+s_B^z)$ corresponding to the μth eigenvalue ξ_μ, and λ_μ is given by $\lambda_\mu=\lambda+\xi_\mu$. $H_E^{\lambda_\mu}$ is given from H_E by replacement of λ with λ_μ. To examine the effect of the environment on the dynamical decoherence of two-qubit state, we should obtain the time evolution operator $U(t)=\exp(-iHt)$. To get an analytical result, we follow the standard procedure by defining the conventional Jordan-Wigner transformation which maps spins to one-dimensional spinless fermions with creation (annihilation) operators $c_l^\dagger$ (c_l) [30]. After a straightforward deduction, the dressed environmental Hamiltonian becomes

$$H_E^{(\lambda_\mu)} = \sum_{l}^{L}[(1+2iD)c_l^\dagger c_{l+1} + (1-2iD)c_{l+1}^\dagger c_l + \gamma(c_l^\dagger c_{l+1}^\dagger + c_{l+1}c_l) + \lambda_\mu(1-2c_l^\dagger c_l)]. \tag{3}$$

Now by introducing the Fourier transforms of the fermionic operators described by $d_k=\frac{1}{\sqrt{L}}\Sigma_l c_l e^{-i2\pi lk/L}$, Hamiltonian (1) can be diagonalized by transforming the fermion operators to momentum space and then using the Bogoliubov transformation. The result is [30,34]

$$H_k^{(\lambda_\mu)} = \sum_k \Lambda_k^{(\lambda_\mu)}\left(\beta_{k,\lambda_\mu}^\dagger\beta_{k,\lambda_\mu} - \frac{1}{2}\right), \tag{4}$$

where the energy spectrum $\Lambda_k^{(\lambda_\mu)}$ is expressed by [35]

$$\Lambda_k^{(\lambda_\mu)} = 2\left[\varepsilon_k + 2D\sin\left(\frac{2\pi k}{L}\right)\right] \tag{5}$$

with $\varepsilon_k=\sqrt{[\cos(\frac{2\pi k}{L})-\lambda_\mu]^2+\gamma^2\sin^2(\frac{2\pi k}{L})}$ and the corresponding Bogoliubov transformed fermion operators are defined by

$$\beta_{k,\lambda_\mu} = \cos\frac{\theta_k^{(\lambda_\mu)}}{2}d_k - i\sin\frac{\theta_k^{(\lambda_\mu)}}{2}d_{-k}^\dagger \tag{6}$$

with angles $\theta_k^{(\lambda_\mu)}$ satisfying

$$\theta_k^{(\lambda_\mu)} = \arctan\left[\frac{\gamma\sin\frac{2\pi k}{L}}{\lambda_\mu - \cos\frac{2\pi k}{L}}\right]. \tag{7}$$

Here it should be mentioned that the energy spectrum derived in Ref. [34] may not be correct. In terms of these notations, we can drive the time evolution of quantum states and obtain the decoherence dynamics induced by the interaction between the system and the environment. Let the initial state be $|\Psi(0)\rangle=|\Phi(0)\rangle_{AB}\otimes|\Phi(0)\rangle_E$, where $|\Phi(0)\rangle_{AB}$ and $|\Phi(0)\rangle_E$ are the initial states of the two central spins and the environment spin chain ones, respectively. So $|\Psi(t)\rangle=U(t)|\Psi(0)\rangle$, where $U(t)=\exp(-iHt)$. Then the reduced density matrix can be obtained with following equation:

$$\rho_{A,B}(t) = \mathrm{Tr}_E|\Psi(t)\rangle\langle\Psi(t)| = \sum_{\mu,\nu=1}^{\zeta} c_\mu c_\nu^* \langle\Phi_E|U_E^{\dagger(\lambda_\nu)}(t)U_E^{(\lambda_\mu)}(t)|\Phi_E\rangle|\mu\rangle\langle\nu|, \tag{8}$$

where $c_\mu=\langle\phi_\mu|\Phi_{AB}\rangle$ and $U_E^{(\lambda_\mu)}(t)=\exp(-iH_E^{(\lambda_\mu)}t)$ is the projected time evolution operator for the spin chain dressed by the system-environment interaction parameter λ_μ. We define the decoherence factor $|F(t)|$ as follows:

$$F(t) = \langle\Phi_E|U_E^{\dagger(\lambda_\nu)}(t)U_E^{(\lambda_\mu)}(t)|\Phi_E\rangle. \tag{9}$$

Obviously, when $\mu=\nu$, $|F(t)_{\mu\nu}|$ equals to 1. $|F(t)_{\mu\nu}|$ can be considered as the amplitude of the overlap of different bases of the system under the defined environment.

III. DYNAMICAL DECOHERENCE OF TWO QUBITS

First, we investigate the dynamic evolution of decoherence factor for a two-qubit case, so $\lambda_1=\lambda+g$, $\lambda_2=\lambda-g$,

$\lambda_3=\lambda$, and $\lambda_4=\lambda$. We assume that the two qubits initially stem from a maximally entangled state,

$$|\Phi\rangle_{AB}=\frac{1}{\sqrt{2}}(|00\rangle_{AB}+|11\rangle_{AB}), \tag{10}$$

where $|0\rangle$ and $|1\rangle$ denote the spins up and down, respectively. From Eq. (8), in the basis spanned by $\{|00\rangle,|11\rangle,|01\rangle,|10\rangle\}$, the reduced density matrix of the two-qubit system is obtained as [26]

$$\rho_{A,B}=\frac{1}{2}\begin{pmatrix}1 & F(t)\\ F^*(t) & 1\end{pmatrix}\oplus Z_{2\times 2}, \tag{11}$$

where $F(t)$ is the decoherence factor defined by Eq. (9) and $Z_{2\times 2}$ denotes the 2×2 zero matrix.

The initial state of environment $|\Phi(0)\rangle_E$ is assumed to be $|G\rangle_\lambda=\Pi_{k=1}^{M}(\cos\frac{\theta_k^\lambda}{2}|0\rangle_k|0\rangle_{-k}+i\sin\frac{\theta_k^\lambda}{2}|1\rangle_k|1\rangle_{-k})$, where $|0\rangle_k$ and $|1\rangle_k$ denote the vacuum and single excitation of the kth mode d_k, respectively. Noticing that

$$\beta_{k,\lambda_\mu}=(\cos\Theta_k^{(\lambda_\mu)})\beta_{k,\lambda}-i(\sin\Theta_k^{(\lambda_\mu)})\beta^\dagger_{-k,\lambda}, \tag{12}$$

where $\Theta_k^{(\lambda_\mu)}=(\theta_k^{(\lambda_\mu)}-\theta_k^{(\lambda)})/2$ and $\beta_{k,\lambda_\mu},\beta_{k,\lambda}$ are the normal mode dressed by the system-environment interaction and the purely environment, respectively, the ground state $|G\rangle_\lambda$ of the Hamiltonian H_E can be obtained from the ground state $|G\rangle_{\lambda_\mu}$ of the qubit-dressed Hamiltonian $H_E^{\lambda_\mu}$ by the following relation [23–25]:

$$|G\rangle_\lambda=\prod_{k=1}^{M}(\cos\Theta_k^{(\lambda_\mu)}+i\sin\Theta_k^{(\lambda_\mu)}\beta^\dagger_{k,\lambda_\mu}\beta^\dagger_{-k,\lambda_\mu})|G\rangle_{\lambda_\mu}. \tag{13}$$

Then the decoherence factor $|F(t)|$ can be obtained from Eqs. (9) and (13) as follows [29]:

$$\begin{aligned}|F(t)|=|{}_\lambda\langle G|U_E^{\dagger(\lambda_2)}(t)U_E^{(\lambda_1)}(t)|G\rangle_\lambda|=\prod_{k>0}\{&1+2\sin(2\Theta_k^{(\lambda_1)})\sin(2\Theta_k^{(\lambda_2)})\sin(\Lambda_k^{(\lambda_1)}t)\sin(\Lambda_k^{(\lambda_2)}t)\cos(\Lambda_k^{(\lambda_1)}t-\Lambda_k^{(\lambda_2)}t)\\ &-4\sin(2\Theta_k^{(\lambda_1)})\sin(2\Theta_k^{(\lambda_2)})\sin^2(\Theta_k^{(\lambda_1)}-\Theta_k^{(\lambda_2)})\sin^2(\Lambda_k^{(\lambda_1)}t)\sin^2(\Lambda_k^{(\lambda_2)}t)\\ &-\sin^2(2\Theta_k^{(\lambda_1)})\sin^2(\Lambda_k^{(\lambda_1)}t)-\sin^2(2\Theta_k^{(\lambda_2)})\sin^2(\Lambda_k^{(\lambda_2)}t)\}^{1/2}\\ \equiv\prod_{k>0}F_k(t).&\end{aligned} \tag{14}$$

A. Weak-coupling case ($g\ll 1$)

Before proceeding with the detailed calculation, we first consult to earlier work on relatively simpler system. Yuan *et al.* [29] studied the decoherence factor of two qubits coupled to an Ising chain under the weak-coupling case. They found that apart from the critical point λ_c, $|F(t)|$ in the time domain is characterized by an oscillatory localization behavior. When λ approaches λ_c, $|F(t)|$ evolves from unity to zero in a very short time, which implies that the disentanglement of the two central spins is best enhanced by a QPT of the environmental spin chain. Now, we consider the effect of DM interaction on the decay of the decoherence factor for the two-qubit interacting with the environment in the weak-coupling region. First, we investigate the dynamical property of $F(t)$ by numerical calculation from the exact expression (14). In Fig. 1, $|F(t)|$ is plotted as a function of parameter D and time t with parameters $L=801$, $g=0.05$, and $\gamma=1.0$. At point $\lambda=1$, we vary the value of D from 0 to 1.0. It can be found that the decoherence factor decays more sharply with increasing the intensity of DM interaction in a short-time regime. However, the plot also indicates that the effect of DM interaction on the decay of decoherence factor is not remarkable. To check this phenomenon, we carry out an analysis similar to Ref. [23]. We introduce a cutoff number K_c and define the partial product for the decoherence factor,

$$|F(t)|_c=\prod_{k>0}^{K_c}F_k(t)\geq|F(t)|, \tag{15}$$

from which the corresponding partial sum is obtained as

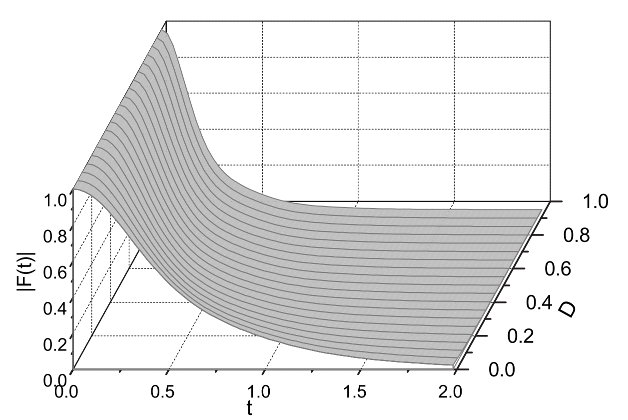

FIG. 1. Decoherence factor $|F(t)|$ as a function of D and time t for two qubits coupled to an Ising spin chain with DM interaction in the weak-coupling region under parameters $\lambda=1.0$, $\gamma=1.0$, $g=0.05$, and $L=801$. The unit of quantity t is $\hbar/J$.

$$S(t) = \ln|F(t)|_c = -\sum_{k>0}^{K_c} |\ln F_k|. \tag{16}$$

For the case of small k and large L, we have

$$\Lambda_k^{(\lambda)} \approx 2|\lambda - 1| + 4D \sin\left(\frac{2\pi k}{L}\right),$$

$$\Lambda_k^{(\lambda_\mu)} \approx 2|\lambda_\mu - 1| + 4D \sin\left(\frac{2\pi k}{L}\right), \tag{17}$$

and then

$$\sin(2\Theta_k^{(\lambda_\mu)}) \approx \frac{\mp 2\gamma\pi k g}{L|(\lambda_\mu - 1)(\lambda - 1)|},$$

$$\sin(\Theta_k^{(\lambda_1)} - \Theta_k^{(\lambda_2)}) \approx \frac{-2\gamma\pi k g}{L|(\lambda_1 - 1)(\lambda_2 - 1)|}. \tag{18}$$

As a result, if L is large enough and k is small relatively, the approximation of $S(t)$ can be obtained as

$$S(t) \approx -\frac{1}{2}\frac{4\pi^2\gamma^2 g^2}{L^2(\lambda - 1)^2}\sum_k^{K_c} k^2\left\{\frac{\sin^2(\Lambda_k^{(\lambda_2)}t)}{(\lambda_2 - 1)^2} + \frac{\sin^2(\Lambda_k^{(\lambda_1)}t)}{(\lambda_1 - 1)^2} + 2\frac{\sin(\Lambda_k^{(\lambda_1)}t)\sin(\Lambda_k^{(\lambda_2)}t)}{|(\lambda_1 - 1)(\lambda_2 - 1)|}\cos(4gt)\right\}. \tag{19}$$

In the derivation of the above equation, we ignore the terms related to the sum k^4/L^4 and employ the approximation formula $\ln(1-x) \approx -x$ for small x. Consequently, in a short time and when $\lambda \to 1$, one has

$$|F(t)| \approx e^{-(\tau_1+\tau_2)t^2}, \tag{20}$$

where

$$\tau_1 = 8E(K)\gamma^2 g^2/(\lambda - 1)^2, \quad E(K) = 4\pi^2\sum_{k=1}^{K_c} k^2/L^2,$$

$$\tau_2 = 32F(K)\gamma^2 g D/(\lambda - 1)^2, \quad F(K) = 8\pi^3\sum_{k=1}^{K} k^3/L^3. \tag{21}$$

From the above approximation analysis we may expect that when the parameter λ is adjusted to point $\lambda = 1$, the decoherence factor will exponentially decay with the second power of time. From Eqs. (20) and (21), it is easily seen that the decoherence factor is determined by τ_1 and τ_2; but from Eq. (21), we can see that $E(K_c) \gg F(K_c)$. So $F(t)$ is determined mainly by τ_1, and D only can change the decay slightly in the weak-coupling region. This is consistent with our numerical results shown in Fig. 1.

We give a brief explanation about the effect of DM interaction on the decoherence factor under the case of $\gamma \neq 1$ which means a general XY spin chain. In Figs. 2(a)–2(c) we plot the decoherence factor against time t under different D with γ equal to 0.8, 0.6, and 0.4, respectively. We can see that the decay of the decoherence factor can be enhanced with increasing D. Furthermore, Fig. 2 shows also that the smaller the values of γ, the stronger the effect of the DM interaction on the decay of decoherence factor.

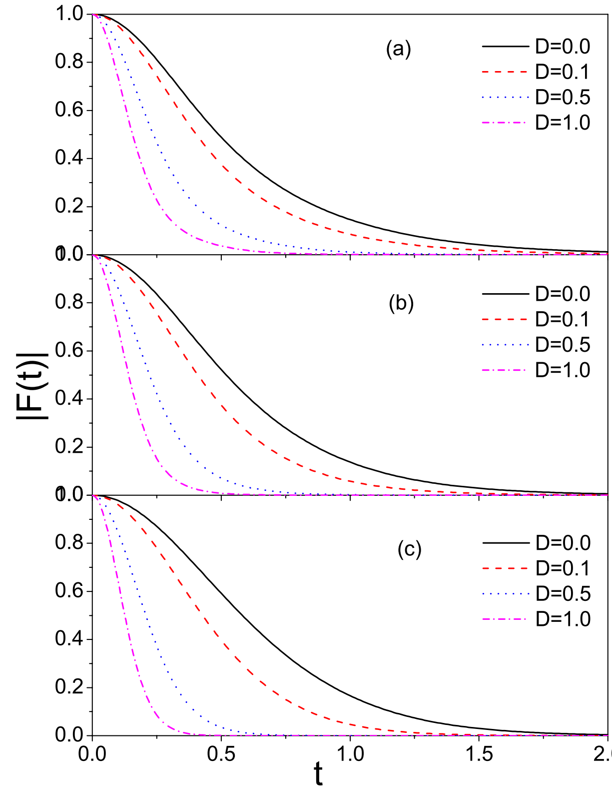

FIG. 2. (Color online) Decoherence factor $|F(t)|$ as a function of time t under different D for two qubits coupled to a general XY spin chain with DM interaction in the weak-coupling region under parameters $\lambda = 1.0$, $g = 0.05$, and $L = 801$. (a) $\gamma = 0.8$, (b) $\gamma = 0.6$, and (c) $\gamma = 0.4$. The unit of quantity t is $\hbar/J$.

B. Strong-coupling case ($g \gg 1$)

We now turn to discuss the effect of DM interaction on the decoherence factor in the strong-coupling regime $g \gg 1$. In a previous work, Yuan *et al.* [29] revealed that the decay of $|F(t)|$ is characterized by an oscillatory Gaussian envelope for a pure Ising spin chain environment. In Fig. 3 we display the time evolution of $|F(t)|$ as a function of time t with the parameters $\lambda = 1$, $\gamma = 1$, $L = 801$, and $g = 500$. It is observed that the decay is characterized by an oscillatory Gaussian envelop, too. However, in contrast to the pure Ising chain environment, the width of the Gaussian envelop is very sensitive to the DM interaction D. In other words, the decay of the decoherence factor can be enhanced dramatically by the DM interaction. To explain this feature, we take a similar tactic used by Yuan *et al.* [29]. When $g \gg 1$, from the angle for Bogoliubov transformation, $\theta_k^{(\lambda_\mu)} = \arctan[\frac{\gamma \sin\frac{2\pi k}{L}}{\lambda_\mu - \cos\frac{2\pi k}{L}}]$, we have $\theta_k^{(\lambda_1)} \approx 0$ and $\theta_k^{(\lambda_2)} \approx \pi$, so we get

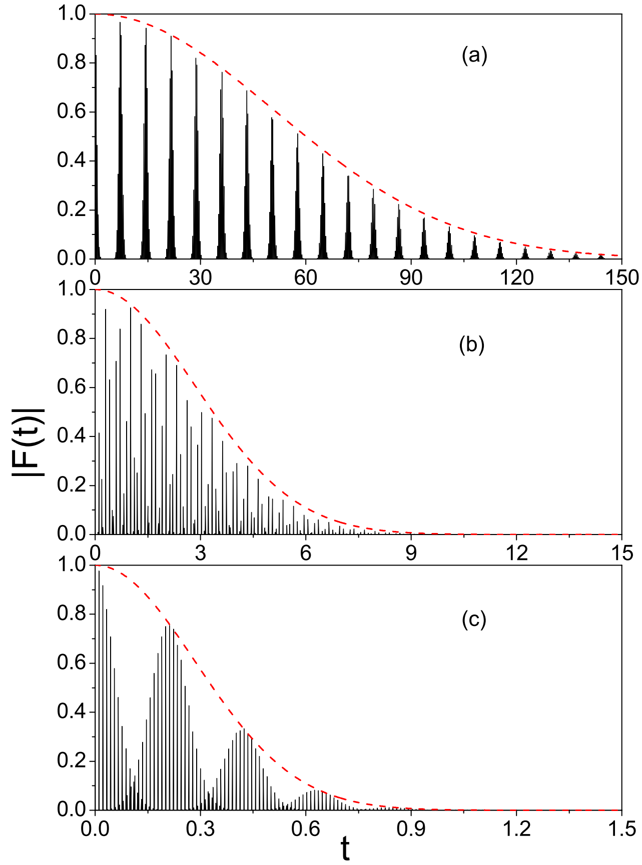

FIG. 3. (Color online) Decoherence factor $|F(t)|$ as a function of the time t for two central spin qubits coupled to an Ising spin chain under different D in the strong-coupling region under parameters $\lambda=1.0$, $\gamma=1.0$, $g=500$, and $L=801$. (a) $D=0.0$, (b) $D=0.01$, and (c) $D=0.1$. The unit of quantity t is $\hbar/J$.

$$|F(t)| \approx \prod_k |\cos^2 \Theta_k^{(\lambda_1)} \exp(i\Lambda_k t) + \sin^2 \Theta_k^{(\lambda_1)} \exp(-i\Lambda_k t)|, \tag{22}$$

where $\Lambda_k=\Lambda_k^{(\lambda_1)}+\Lambda_k^{(\lambda_2)}$. This expression is similar to that given in Refs. [21,24]. We can follow the mathematical procedure and the resultant approximate expression for $|F(t)|$ is as follows:

$$|F(t)| \approx \exp(-s_L^2 t^2/2)|\cos(\Lambda t)|^{L/2}, \tag{23}$$

where Λ is the mean value of Λ_k.

$$\begin{aligned}\Lambda &= \frac{1}{M}\sum_{k>0} (\Lambda_k^{\lambda_1} + \Lambda_k^{\lambda_2}) \\ &\approx \frac{1}{M}\sum_{k>0} 2\left(2\sqrt{g^2+\gamma^2 \sin^2\frac{2\pi k}{L}} + 4D \sin\frac{2\pi k}{L}\right) \\ &\approx \frac{1}{M}\sum_{k>0} 4\left(g + \frac{1}{2}\frac{\gamma^2}{g}\sin^2\frac{2\pi k}{L}\right) \\ &\approx 4g + \frac{\gamma^2}{g}.\end{aligned} \tag{24}$$

In the deduction, the approximation formula $(1-x)^{1/2}\approx 1-(1/2)x$ is employed for small x and

$$s_L^2 = \sum_{k>0} \sin^2(2\Theta_k^{(\lambda_1)})\delta_k^2. \tag{25}$$

Here the quantity δ_k describes the deviation of Λ_k from its mean values. Its value is

$$\begin{aligned}\delta_k = \Lambda_k - \Lambda &\approx 2\left(2\sqrt{g^2+\gamma^2 \sin^2\frac{2\pi k}{L}} + 4D \sin\frac{2\pi k}{L}\right) - 4g \\ &- \frac{\gamma^2}{g} \approx -\frac{\gamma^2}{g}\cos\frac{4\pi k}{L} + 8D \sin\frac{2\pi k}{L}.\end{aligned} \tag{26}$$

So one can see that the width of the Gaussian envelope is proportional to $\{(\frac{\gamma^4}{g^2}+64D^2)L\}^{-1/2}$. Figures 3(a)–3(c) show the numerical results plotted by solid line and the Gaussian envelope factor plotted by dashed ones for the intensity of DM interaction D is equal to 0.0, 0.01, and 0.1, respectively. From the above discussion, we can conclude that the decay of the decoherence factor is determined mainly by D in the region of $g \gg 1$.

IV. DYNAMICAL DECOHERENCE OF TWO QUTRITS

Now, we investigate the dynamic evolution of the decoherence factor for a two-qutrit case and assume that the two-qutrit case initially stems from a maximally entangled state,

$$|\Phi\rangle_{AB} = \frac{1}{\sqrt{3}}(|00\rangle_{AB} + |11\rangle_{AB} + |22\rangle_{AB}), \tag{27}$$

where $|0\rangle$, $|1\rangle$, and $|2\rangle$ denote the spin 1 with magnetic quantum numbers 1, 0, and -1, respectively. From Eq. (8), in the basis spanned by $\{|00\rangle,|11\rangle,|22\rangle,|01\rangle,|10\rangle,|02\rangle,|20\rangle,|12\rangle,|21\rangle\}$, the reduced density matrix of the two-qutrit system is [26]

$$\rho_{A,B} = \frac{1}{3}\begin{pmatrix} 1 & F_{12}(t) & F_{13}(t) \\ F_{12}^*(t) & 1 & F_{23}(t) \\ F_{13}^*(t) & F_{23}^*(t) & 1 \end{pmatrix} \oplus Z_{2\times 2} \oplus Z_{2\times 2} \oplus Z_{2\times 2}, \tag{28}$$

where the decoherence factors $F_{12}(t)$, $F_{13}(t)$, and $F_{23}(t)$ are given by Eq. (14) exactly and λ_μ ($\mu=1,2,3$) in $|F_{\mu\nu}(t)|$ can be obtained by substituting $\lambda_1=\lambda+g$, $\lambda_2=\lambda$, and $\lambda_3=\lambda-g$. From the previous discussion, it is obvious that the property of $|F_{13}(t)|$ is completely similar to the two-qubit case both in the weak-coupling region and the strong one. In the following, we only discuss the role of DM interaction on the decoherence factors $|F_{12}(t)|$ and $|F_{23}(t)|$.

A. Weak-coupling case ($g \ll 1$)

We first investigate the effect of DM interaction on the decoherence factor $|F_{12(23)}(t)|$ in the weak-coupling region. From $\Theta_k^{(\lambda_\mu)}=(\theta_k^{(\lambda_\mu)}-\theta_k^{(\lambda)})/2$, Eq. (14) can be rewritten as

$$|F_{12(23)}(t)| = \prod_{k>0} \{1 - \sin^2(2\Theta_k^{\lambda_{1(3)}})\sin^2(\Lambda_k^{\lambda_{1(3)}}t)\}^{1/2}. \tag{29}$$

So, one can get $S(t)$ as follows:

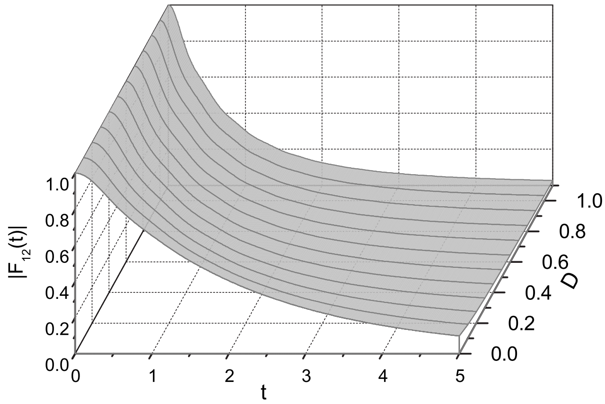

FIG. 4. Decoherence $|F_{12}(t)|$ as a function of D and time t for two qutrits coupled to an Ising spin chain with DM interaction in the weak-coupling region under the parameters $\lambda=1.0$, $\gamma=1.0$, $g=0.05$, and $L=801$. The unit of quantity t is $\hbar/J$.

$$S(t) \approx -\frac{1}{2}\frac{4\pi^2\gamma^2 g^2}{L^2(\lambda-1)^2}\sum_{k}^{K_c}\left\{k^2\frac{\sin(\Lambda_k^{\lambda_{1(3)}}t)}{(\lambda_{1(3)}-1)}\right\}^2. \tag{30}$$

Consequently, in a short time and when $\lambda\to 1$, one has the similar previous result, $|F_{12(23)}|\approx e^{-(\tau_1+\tau_2)t^2}$, where $\tau_1=2E(K)\gamma^2 g^2/(\lambda-1)^2$ and $\tau_2=8F(K)\gamma^2 gD/(\lambda-1)^2$. $E(K)$ and $F(K)$ are completely the previous ones. One can see that, when the parameter λ approaches point $\lambda=1$, the decoherence factor will exponentially decay with the second power of time, too. We can find that the decay of $|F_{12(23)}(t)|$ can be enhanced slightly by the DM interaction, and the numerical results are shown in Fig. 4.

B. Strong-coupling case ($g\gg 1$)

Now we check the dynamical property of $|F_{12(23)}(t)|$ by numerical analysis from the exact expression (14). In Fig. 5, $|F_{12}(t)|$ is plotted as a function of time t and D. Panels (a), (b), and (c) correspond to parameter D equal to 0.0, 1.0, and 2.0, respectively. We find that the decay of the decoherence factor approaches zero sharply in the strong-coupling region and characterized by an oscillatory Gaussian envelop, too. Despite the DM interaction can enhance the decay of the decoherence factor, however, in contrast to the two-qubit case, the decays of the decoherence factors $|F_{12}(t)|$ and $|F_{23}(t)|$ are not significantly affected by the DM interaction.

V. CONCLUSION

In summary, we have studied the dynamic process of the decoherence factor of two spin qubits coupled to a general *XY* spin chain with DM interaction. In the weak-coupling region ($g\ll 1$), the decoherence factor presents a Gaussian behavior in a short time, e.g., $|F(t)|=e^{-(\tau_1+\tau_2)t^2}$, and the DM interaction can enhance the decay of the decoherence factor slightly. However, in the strong-coupling region, we find that the decay of the decoherence factor is characterized by an oscillatory Gaussian envelop, and the width of the Gaussian envelop is very sensitive to the DM interaction. It is mainly determined by the values of D and is proportional to $\{(\frac{\gamma^4}{g^2}+64D^2)L\}^{-1/2}$. The role of DM interaction on decoherence factor for a general *XY* chain environment is also analyzed. In the weak-coupling region, DM interaction can enhance the decay of the decoherence, too.

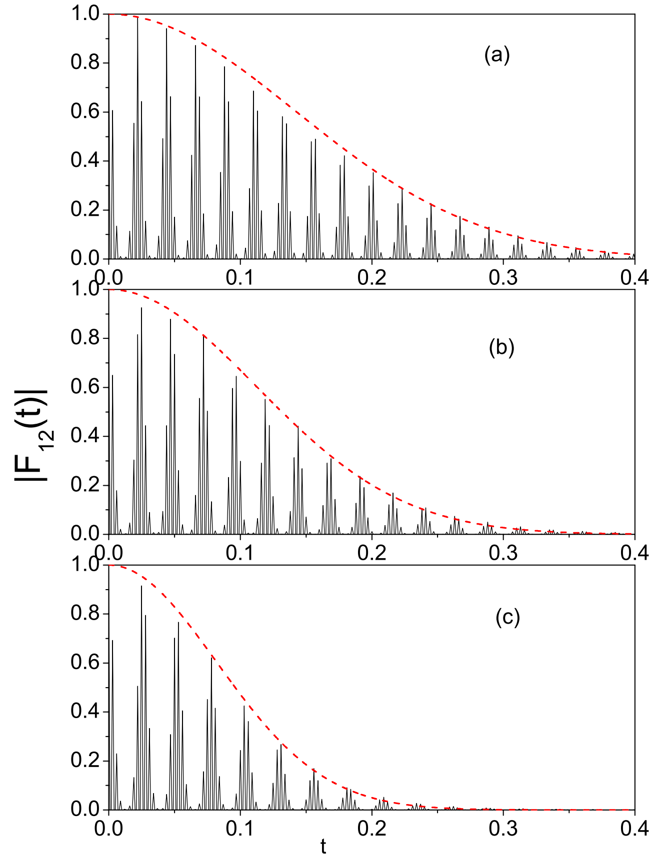

FIG. 5. (Color online) Decoherence factors $|F_{12}(t)|$ as a function of the time t for two qutrits coupled to an Ising spin chain under different D in the strong-coupling region under the parameters $\lambda=1.0$, $\gamma=1.0$, $g=500$, and $L=201$. (a) $D=0.0$, (b) $D=1.0$, and (c) $D=2.0$. The unit of quantity t is $\hbar/J$.

In such case, the decoherence factor $|F_{13}(t)|$ exhibits the same behavior as the two-qubit case. However, in the strong-coupling region, the decays of $|F_{12}(t)|$ and $|F_{23}(t)|$ approach zero sharply, in contrast to the two-qubit case, which are not significantly affected by the DM interaction.

ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support from the National Natural Science Foundation of China (Grant Nos. 10674061, 50832002, and 50572038), the National Key Projects for Basic Researches of China (Grant Nos. 2006CB921802 and 2009CB623303), and the Key Projects of Hubei Province (Grant No. D20092204).

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