Disentanglement from spin environment: Role of multisite interaction

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The dynamical entanglement of two spin-qubits coupled to an XY spin-chain with a three-site interaction environment is studied. The dynamical process of disentanglement is numerically and analytically investigated. In the strong-coupling region, the three-site interaction can dramatically accelerate the decay of the entanglement between the two spin-qubits. However, in the weak-coupling region, the role of this interaction in the process of disentanglement depends on the strength of the three-site interaction; for example, in some specific intervals, this interaction can remarkably delay the process of disentanglement.

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As one of the most essential features in quantum theory, quantum entanglement plays a key role as a potential resource for quantum computation and quantum information processing, and has attracted much attention in many branches of physics, both theoretical and experimental [1–3]. On the other hand, a real physical system can never be isolated; the interaction between the quantum system and its environment leads to decoherence [4]. Consequently, the initial pure state of the system may quickly decay into an incoherent mixture of many states, which is a major obstacle for building a practical quantum processor. This complex quantum many-body phenomenon is challenging, and many works have dealt with various models for both correlated and uncorrelated environments [5–14]. Typical examples include the decoherence from a spin environment consisting of N independent spins [5], the dynamical entanglement under the background of pure vacuum noise [6], the decoherence induced by a chaotic many-spin bath [11,12], as well as the electron-spin decoherence in contact with a mesoscopic bath of many interacting nuclear spins in an InAs quantum dot (QD) [14]. Furthermore, the decoherence of a system coupled to a spin environment with quantum phase transition (QPT) [15–19], occurring at zero temperature and driven only by quantum fluctuations [20], was investigated. For a quantum two-level system coupled to an Ising chain, it was found that the decaying behavior of the Loschmidt echo (LE) is best enhanced by the QPT of the surrounding system [15]. It was also demonstrated that the decay of quantum coherence may be a universal feature of when the system coupling drives a QPT in the environment [16].

Recently, several experiments evidenced that the entanglement can exist in variety of macroscopic and strongly correlated solids [21,22], and some magnetic compounds can be used as the entanglement reservoir (e.g., $Na_2Cu_5Si_4O_{14}$ [23] and $MgMnB_2O_5$ [24]). These magnetic materials can be understood in the framework of variational spin-chain models, which allows a model investigation of the entanglement in such materials. Many theoretical studies paid attention not only to the nearest-neighbor spin-spin interactions but also to the next-nearest-neighbor ones, as well as multiple spin-exchange models. In comparison with the models with only nearest-neighbor interactions, these more complicated models seem closer to a real situation [25]. For example, the additional terms involving the multisite interaction operators were recognized to be important for the description of many physical systems (e.g., the solid ^{3}He [26]). In previous works related to the decoherence due to the spin environment, these important and realistic interactions are rarely considered despite the fact that they are often present in a real spin system. In this work, the goal is to determine how the multisite interaction affects the process of disentanglement.

We choose the environment to be an XY spin-chain with $XZY-YZX$ type of three-site interaction. The corresponding Hamiltonian reads

$$\begin{aligned} H = &-\sum_{l}^{L} J\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right) \\ &-\sum_{l}^{L}\alpha J\left(\sigma_{l-1}^x\sigma_l^z\sigma_{l+1}^y - \sigma_{l-1}^y\sigma_l^z\sigma_{l+1}^x\right) \\ &-\sum_{l}^{L}\frac{J}{2}\left(g_l^A\sigma_A^z + g_l^B\sigma_B^z\right)\sigma_l^z = H_E + H_I. \end{aligned} \quad (1)$$

where the spin-coupling interaction in units of J is unit, α and λ are dimensionless, and $g_l^{A(B)}$ denotes the coupling strength between the chain and the two spin-qubits. To expediently obtain an analytical result, we assume $g_l^A = g_l^B = g$. Because $[\sigma_A^z + \sigma_B^z, \sigma_l^\beta] = 0$, the operator $\sigma_A^z + \sigma_B^z$, which is a conserved quantity, can be treated as a constant with different values corresponding to its eigenvalues in the two-spin subspace: $\varsigma_1 = g$, $\varsigma_2 = -g$, $\varsigma_3 = \varsigma_4 = 0$. The corresponding eigenvectors can be written as $|\varphi_1\rangle = |00\rangle$, $|\varphi_2\rangle = |11\rangle$, $|\varphi_{3(4)}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$. So Eq. (1) can be rewritten as $H = \sum_{\mu=1}^{\varsigma} |\varphi_\mu\rangle\langle\varphi_\mu| \otimes H_E^{\lambda_\mu}$, where $\lambda_\mu = \lambda + \varsigma_\mu$, and $H_E^{\lambda_\mu}$ is given from H_E by replacing λ with λ_μ.

By combining the Jordan-Wigner transformation [20] and the Fourier transforms of the fermionic operators described by $d_k = \frac{1}{\sqrt{L}}\sum_l a_l e^{-i2\pi lk/L}$, and then using the Bogoliubov transformation $\gamma_{k,\lambda_\mu} = \cos\frac{\theta_k^{\lambda_\mu}}{2} d_k - i\sin\frac{\theta_k^{\lambda_\mu}}{2} d_{-k}^\dagger$ with $\tan(\theta_k^{\lambda_\mu}) = \gamma\sin(\frac{2\pi k}{L})/[\lambda_\mu - \cos(\frac{2\pi k}{L})]$, the Hamiltonian can be diagonalized as $H_E^{\lambda_\mu} = \sum_k \epsilon_k^{\lambda_\mu}(\gamma_{k,\lambda_\mu}^\dagger \gamma_{k,\lambda_\mu} - \frac{1}{2})$, where $\epsilon_k^{\lambda_\mu} = 2[\varepsilon_k + \alpha \sin(\frac{4\pi k}{L})]$ with $\varepsilon_k = \{[\cos(\frac{2\pi k}{L}) - \lambda_\mu]^2 + \gamma^2 \sin^2(\frac{2\pi k}{L})\}^{\frac{1}{2}}$.

Let the initial state of the composite system be $|\Psi(0)\rangle = |\Phi(0)\rangle_{AB} \otimes |\Phi(0)\rangle_E$, where $|\Phi(0)\rangle_{AB}$ and $|\Phi(0)\rangle_E$ are the initial states of the two central spins and the environment spin-chain ones, respectively. We have $|\Psi(t)\rangle = U(t)|\Psi(0)\rangle$, where $U(t) = \exp(-iHt)$. The reduced density matrix can be obtained as [17,18] $\rho_{A,B}(t) = \sum_{\mu,\nu}^{\zeta} c_\mu c_\nu^* \langle \Phi_E | U_E^{\lambda_\nu \dagger}(t) U_E^{\lambda_\mu}(t) | \Phi_E \rangle |\mu\rangle\langle\nu|$, where $c_\mu = \langle \varphi_\mu | \Phi_{AB} \rangle$, and $U_E^{\lambda_\mu}(t) = \exp(-iH_E^{\lambda_\mu} t)$ is the projected time-evolution operator for the spin-chain dressed by the system-environment interaction parameter λ_μ. The off-diagonal elements of the reduced density matrix are defined by the decoherence factor $F(t)$:

$$F(t) = \langle \Phi_E | U_E^{\lambda_\nu \dagger}(t) U_E^{\lambda_\mu}(t) | \Phi_E \rangle. \tag{2}$$

It is noteworthy that the reduced density matrix $\rho_{AB}(t)$ depends on the initial state of the central two spin-qubits Φ_{AB} as well as the environmental spin-chain Φ_E. It is obvious that $F(t) \equiv 1$ if $|\Phi_{AB}(0)\rangle$ lies in the subspace spanned by $|\varphi_{3(4)}\rangle$. We assume that the two spin-qubits initially stem from a maximally entangled state $|\Phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$. In the basis spanned by $\{|00\rangle, |11\rangle, |01\rangle, |10\rangle\}$, the reduced density matrix of the two-qubit system is expressed as [17]

$$\rho_{A,B}(t) = \frac{1}{2}\begin{pmatrix} 1 & F(t) \\ F^*(t) & 1 \end{pmatrix} \oplus Z_{2\times 2}, \tag{3}$$

where $F(t)$ is the decoherence factor defined by Eq. (2), and $Z_{2\times 2}$ denotes the 2×2 zero matrix. Now, the concurrence [27] of the reduced density matrix can be obtained as $\mathcal{C}(t) = \mathcal{C}_0 |F(t)|$ [17], which is proportional to the norm of the decoherence factor, and $\mathcal{C}_0$ is the concurrence of the initial state and obviously equals 1.

In earlier works, the thermal equilibrium state at finite temperature was occasionally chosen as the initial environment [e.g., $\rho_E(0) = \frac{1}{Z}\exp(-\beta \sum_k \epsilon_k^\lambda \gamma_{k,\lambda}^\dagger \gamma_{k,\lambda})$]. When the temperature is quite low, Zanardi *et al.* indicated that the decay of the decoherence factor (LE) is very similar to the situation where the initial state stems from the ground state [28]. In this work, we assume that the initial state of the environment $\Phi_E(0)$ is the ground state $|G\rangle_\lambda$ of the pure spin-chain Hamiltonian H_E^λ. This ground state $|G\rangle_\lambda$ is the vacuum of the fermionic modes described by $\gamma_{k,\lambda}|G\rangle_\lambda = 0$ and is given as a tensor product of a qubit-like state, $|G\rangle_\lambda = \otimes_{k=1}^{M}(\cos\frac{\theta_k^\lambda}{2}|0\rangle_k|0\rangle_{-k} + i\sin\frac{\theta_k^\lambda}{2}|1\rangle_k|1\rangle_{-k})$, where $|0\rangle_k$ and $|1\rangle_k$ denote the vacuum and single excitation of the kth mode d_k, respectively. The ground state $|G\rangle_\lambda$ of the Hamiltonian H_E can be obtained from the ground state $|G\rangle_{\lambda_\mu}$ of the qubit-dressed Hamiltonian $H_E^{\lambda_\mu}$ by the relation $|G\rangle_\lambda = \prod_{k=1}^{M}(\cos\Theta_k^{\lambda_\mu} + i\sin\Theta_k^{\lambda_\mu}\gamma_{k,\lambda_\mu}^\dagger \gamma_{-k,\lambda_\mu}^\dagger)|G\rangle_{\lambda_\mu}$, since $\gamma_{k,\lambda_\mu} = (\cos\Theta_k^{\lambda_\mu})\gamma_{k,\lambda} - i(\sin\Theta_k^{\lambda_\mu})\gamma_{-k,\lambda}^\dagger$, where $\Theta_k^{\lambda_\mu} = (\theta_k^{\lambda_\mu} - \theta_k^\lambda)/2$, and $\gamma_{k,\lambda_\mu}, \gamma_{k,\lambda}$ are the normal modes dressed by the system-environment interaction and the pure environment, respectively. Then the concurrence $\mathcal{C}(t)$ can be obtained from Eqs. (2) and (3) [18]:

$$\begin{aligned}\mathcal{C} = \prod_{k>0} \Big[& 1 + 2\sin\left(2\Theta_k^{\lambda_1}\right)\sin\left(2\Theta_k^{\lambda_2}\right)\sin\left(\epsilon_k^{\lambda_1}t\right)\sin\left(\epsilon_k^{\lambda_2}t\right) \\ & \times \cos\left(\epsilon_k^{\lambda_1}t - \epsilon_k^{\lambda_2}t\right) - 4\sin^2\left(\Theta_k^{\lambda_1} - \Theta_k^{\lambda_2}\right)\sin^2\left(\epsilon_k^{\lambda_1}t\right) \\ & \times \sin^2\left(\epsilon_k^{\lambda_2}t\right)\sin\left(2\Theta_k^{\lambda_1}\right)\sin\left(2\Theta_k^{\lambda_2}\right) - \sin^2\left(2\Theta_k^{\lambda_2}\right) \\ & \times \sin^2\left(\epsilon_k^{\lambda_2}t\right) - \sin^2\left(2\Theta_k^{\lambda_1}\right)\sin^2\left(\epsilon_k^{\lambda_1}t\right)\Big]^{\frac{1}{2}}. \end{aligned} \tag{4}$$

Yuan *et al.* showed that the disentanglement of the two spin-qubits is best enhanced by a QPT of the pure XY environment [18]. Now, we consider the effect of the three-site interaction on the decay of the entanglement of the two qubits which are interacting with the environment in the weak-coupling region ($g \ll 1$). First, we investigate the dynamical property $\mathcal{C}(t)$ by numerical calculation from the exact expression of Eq. (4). In Fig. 1, $\mathcal{C}(t)$ is plotted as a function of the three-site interaction intensity α and time t, given parameters $L = 401$, $g = 0.05$, and $\gamma = 1.0$. At the point $\lambda = 1$, we vary α from -1.0 to 0.5. In a short time, it is observed that the concurrence $\mathcal{C}(t)$ decays more sharply with the increasing intensity in the interval $\alpha \geqslant 0$. However, in the region $\alpha < 0$, the decay of the concurrence $\mathcal{C}(t)$ can be delayed remarkably with the increasing intensity along the negative direction. This trend is not monotonous, and the decay becomes serious once more with the increasing intensity of the three-site interaction if α exceeds a certain value. To check this effect, we perform an analysis similar to that done in [15] and introduce a cutoff number K_c and define the partial product for the decoherence factor $|F(t)|_c = \prod_{k>0}^{K_c} F_k(t) \geqslant |F(t)|$, from which the corresponding partial sum is obtained: $S(t) = \ln|F(t)|_c = -\sum_{k>0}^{K_c} |\ln F_k|$. For a weak-coupling parameter ($g \ll 1$) and a large environment size L, $\epsilon_k^\lambda \approx 2|\lambda - 1| + 2\alpha\sin(\frac{4\pi k}{L})$, $\epsilon_k^{\lambda_\mu} \approx 2|\lambda_\mu - 1| + 2\alpha\sin(\frac{4\pi k}{L})$, and then $\sin(2\Theta_k^{\lambda_\mu}) \approx \frac{\mp 2\gamma\pi k g}{L|(\lambda_\mu - 1)(\lambda - 1)|}$, $\sin(\Theta_k^{\lambda_1} - \Theta_k^{\lambda_2}) \approx \frac{-2\gamma\pi k g}{L|(\lambda_1 - 1)(\lambda_2 - 1)|}$. In short time t, we have $S(t) \approx -\frac{1}{2}\frac{4\pi^2\gamma^2 g^2}{L^2(\lambda-1)^2}\sum_k^{K_c} k^2[\frac{\sin^2(\epsilon_k^{\lambda_2}t)}{(\lambda_2-1)^2} + \frac{\sin^2(\epsilon_k^{\lambda_1}t)}{(\lambda_1-1)^2} + 2\frac{\sin(\epsilon_k^{\lambda_1}t)\sin(\epsilon_k^{\lambda_2}t)}{|(\lambda_1-1)(\lambda_2-1)|}]$. As $\lambda \to 1$, one has

$$\mathcal{C}(t) \approx \mathcal{C}_0 e^{-(\tau_1+\tau_2+\tau_3)t^2}, \tag{5}$$

where τ_1, τ_2, and τ_3 are equal to $\frac{8E(K)\gamma^2 g^2}{(\lambda-1)^2}$, $\frac{32F(K)\gamma^2 g\alpha}{(\lambda-1)^2}$, and $\frac{32G(K)\gamma^2\alpha^2}{(\lambda-1)^2}$, respectively; $E(K)$, $F(K)$, and $G(K)$ are equal to

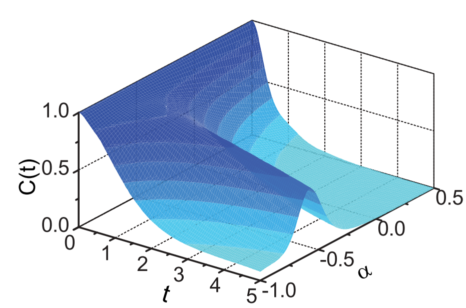

FIG. 1. (Color online) Concurrence $\mathcal{C}$ as a function of α and time t in the weak-coupling region. $\lambda = 1.0$, $\gamma = 1.0$, $g = 0.05$, and $L = 401$. Hereafter, quantity t is in units of $\hbar/J$.

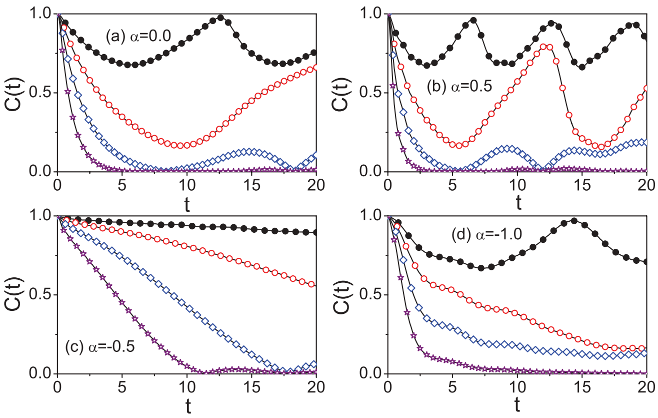

FIG. 2. (Color online) Concurrence $\mathcal{C}$ as a function of time t at different environment sizes in the weak-coupling region. $\lambda = 1.0$, $\gamma = 1.0$, and $g = 0.05$. Here, $L = 51$ (solid circles), $L = 101$ (open circles), $L = 201$ (diamonds), and $L = 401$ (pentagons).

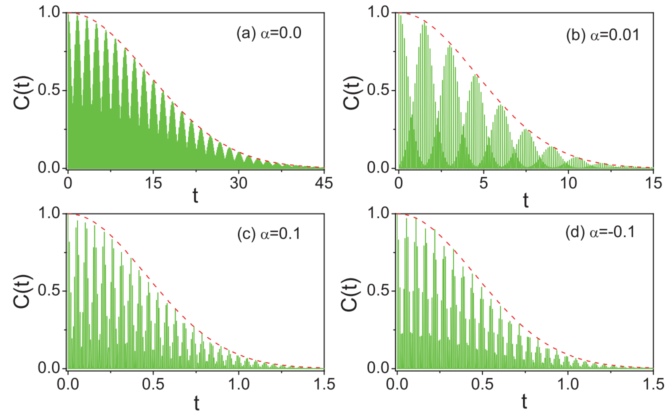

FIG. 3. (Color online) Concurrence $\mathcal{C}$ as a function of time t in the strong-coupling region. $\lambda = 1.0$, $\gamma = 1.0$, $g = 100$, and $L = 401$.

$4\pi^2 \sum(\frac{k}{L})^2$, $8\pi^3 \sum(\frac{k}{L})^3$, and $16\pi^4 \sum(\frac{k}{L})^4$, respectively. The preceding analysis allows us to predict that, when parameter λ is adjusted to the point $\lambda = 1$, the concurrence $\mathcal{C}(t)$ will exponentially decay in the second power of time. From Eq. (5), it is easy to see that the concurrence $\mathcal{C}(t)$ is determined by τ_1, τ_2, and τ_3, while we also can see that the expression $\tau_1 + \tau_2 + \tau_3$ is a second-order polynomial against variable α. This finding indicates the existence of an extremum point at which the decay of the entanglement can be delayed remarkably.

Figure 2 shows the influence of environment size L on the decay behavior of the concurrence. During the process of entanglement evolution, the entanglement is revived several times in the long time region. However. the revivification can be suppressed by increasing L. In the short time region, a large environment size favors a rapid decay of the concurrence in a monotonous manner. This feature can be explained by Eq. (4). Each factor $F_k(t)$ is smaller than unity; thus, it is reasonable to conclude that a larger environment size allows a more effective suppression of the decoherence factor and, consequently, suppression of the entanglement. For $\alpha = -0.5$, we plot the concurrence $\mathcal{C}$ as a function of time t in Fig. 2(c). Comparison with the other three panels shows that the decay of entanglement can be delayed remarkably for all system sizes L in the short time region.

We now turn to investigate the effect of this three-site interaction on the disentanglement of the two spin-qubits in the strong-coupling regime ($g \gg 1$) [29]. Figure 3 plots the time evolution of $\mathcal{C}(t)$ with $g = 100$. It is observed that the decay is characterized by an oscillatory Gaussian envelope. However, in contrast to the pure Ising-chain environment [18], the width of the Gaussian envelope is very sensitive to the three-site interaction α. In other words, the decay of entanglement can be enhanced dramatically by this three-site interaction. Moreover, the plots in Figs. 3(c) and 3(d) indicate that the Gaussian envelope remains unchanged upon variation of α from a positive value to a negative one. To explain these features, we take a similar tactic employed by Yuan [18]. As $g \gg 1$, we have the following from the angle of the Bogoliubov transformation: $\theta_k^{\lambda_1} \approx 0$ and $\theta_k^{\lambda_2} \approx \pi$. Consequently, we obtain [18] $|F(t)| \approx \prod |\cos^2 \Theta_k^{\lambda_1} e^{i\epsilon_k t} + \sin^2 \Theta_k^{\lambda_1} e^{-i\epsilon_k t}|$, where $\epsilon_k = \epsilon_k^{\lambda_1} + \epsilon_k^{\lambda_2}$. This expression is similar to that given in [5]. We can follow the mathematical procedure and derive the approximate expression for $\mathcal{C}(t) \approx \mathcal{C}_0 \exp(-s_L^2 t^2/2)|\cos(\epsilon t)|^{(L-1)/2}$, where ϵ is the mean value of ϵ_k and approximately equal to $4g + \gamma^2/g$ [18,19]; $s_L^2 = \sum \sin^2(2\Theta_k^{\lambda_1})\delta_k^2$, where the quantity δ_k describes the deviation of ϵ_k from its mean values and is approximately equal to $-\frac{\gamma^2}{g}\cos\frac{4\pi k}{L} + 4\alpha \sin\frac{4\pi k}{L}$. In comparison with the pure XY model, this three-site interaction induces the spectral fluctuation of energy ϵ_k more remarkably, which is similar to the situation discussed in [19], where the environment is described by an XY chain with Dzyaloshinskii-Moriya interaction. This is the reason why the three-site interaction can enhance the decay of the entanglement dramatically. Furthermore, one can predict that the width of the Gaussian envelope is proportional to $[(\frac{\gamma^4}{g^2} + 16\alpha^2)L]^{-1/2}$. Figures 3(a)–3(d) show the numerical results (solid lines) and the Gaussian envelope factor (dashed lines) when $\alpha =$ 0.0, 0.01, 0.1, and -0.1, respectively. From the preceding discussion, we can conclude that the decay of the entanglement is determined mainly by α in the region of $g \gg 1$. The width of the envelope is related to α^2, which is consistent with the numerical results shown in Figs. 3(c) and 3(d).

Another interesting topic is the role of this three-site interaction in the dynamical entanglement of the mixed state. Here, we employ the well-known Werner state [30], described by $\rho_{AB}(0) = \frac{1-P}{4} I_{4\times4} + P|\Phi\rangle\langle\Phi|$ as an initial state for the two spin-qubits. Here, $|\Phi\rangle$ is the maximally entangled state given by the previous case, $|\Phi\rangle_{AB}$, $P \in [0, 1]$, and $I_{4\times4}$ denotes a 4×4 unit matrix. We suppose that the initial state of the total system is $\rho(0) = \rho_{AB}(0) \otimes |\Phi_E(0)\rangle\langle\Phi_E(0)|$, where $|\Phi_E(0)\rangle$ is the initial state of the environment described in the previous cases. Then the reduced density matrix for the two-qubit system at time t takes the form [17]

$$\rho_{AB}(t) = \frac{1}{2}\begin{pmatrix} \frac{1+P}{2} & PF(t) \\ PF(t) & \frac{1+P}{2} \end{pmatrix} \oplus \left(\frac{1-P}{4}\right) I_{2\times2}, \qquad (6)$$

and the concurrence can be obtained as $\mathcal{C}(t) = \max\{P[|F(t)| + \frac{1}{2}] - \frac{1}{2}, 0\}$.

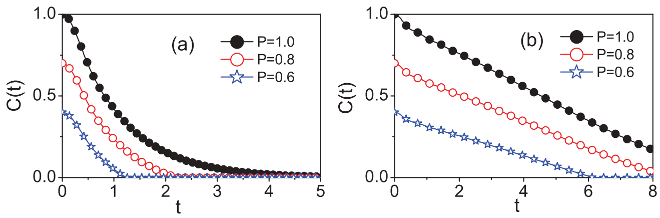

FIG. 4. (Color online) Concurrence $\mathcal{C}$ as a function of time t in the case of the Werner state as the initial state for different P. (a) $\alpha = 0.0$, (b) $\alpha = -0.5$. Here, $\lambda = 1.0$, $\gamma = 1.0$, $g = 0.05$, and $L = 401$.

In Figs. 4(a) and 4(b), the time evolution of the concurrence $\mathcal{C}$ is plotted for three different values of P in the region $\frac{1}{3} < P \leqslant 1$ with weak-coupling strength $g = 0.05$. One finds that the entanglement vanishes abruptly for the mixed states corresponding to $P = 0.6$ and $P = 0.8$ within a finite time. Usually, this finite-time disentanglement is called a "sudden entanglement death," as observed by Yu and Eberly [31]. For the pure-state case ($P = 1$), the entanglement would approach zero asymptotically and takes infinite time. From Fig. 4(b), we can see that the three-site interaction can also remarkably delay the process of disentanglement in the case of the mixed state as an initial state, if proper parameter values are chosen.

In summary, we have studied the process of disentanglement for two spin-qubits coupled to a general XY spin-chain with a three-site interaction environment. In the weak-coupling region, the decay of the entanglement exhibits Gaussian behavior in a short time. Although this three-site interaction can enhance the decay of the two-qubit entanglement in most cases, in some specific intervals the decay of the entanglement can be delayed remarkably. In the strong-coupling region, the decay of the entanglement is characterized by an oscillatory Gaussian envelope, and the envelope width is very sensitive to the three-site interaction. This width is mainly determined by $|\alpha|$. Furthermore, when the two spin-qubits are initially in the mixed state, although the complete disentanglement takes place in a finite time, the additional three-site interaction can still delay the process of disentanglement remarkably.

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[29] For example, in the semiconductor QD system, the electron spin is coupled to the lattice nuclear spins through the contact hyperfine interaction, which has a typical energy scale of 10^6 s^{-1}, while the intrinsic nuclear spin-spin interaction has the nearest-neighbor coupling energy scale 10^2 s^{-1}.
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